Measuring the heat capacity of superfluid ^{4}He in the presence of a heat current near T_λ: progress and prospects

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It has been predicted that the heat capacity of superfluid ^{4}He will diverge strongly at a depressed transition temperature in the presence of a constant heat current, Q. We have built a cell to measure this effect, and have taken preliminary measurements at various heat currents. Our data indicate an enhancement of the heat capacity that varies as a function of Q and diverges at a depressed transition temperature. However, the temperature dependence of our measurements does not agree with previously reported results, leading us to believe that our initial findings were affected by spurious heat flow.

1. INTRODUCTION

There has been significant recent interest in the study of phase transitions in non-equilibrium or dynamic systems. Near the lambda point of ^{4}He, an applied heat flux, Q, creates just such a dynamic situation. According to the two fluid model, a heat current induces a counterflow between the superfluid and the normal fluid, giving the system an extra degree of thermodynamic freedom. It is believed that the presence of superflow depresses the superfluid density.

The existence of a counterflow and consequent reduction in superfluid density depresses the transition temperature and is expected to cause the heat capacity to be enhanced.[1,2] In fact, if the heat current is held constant during the measurement, the superfluid density will become sufficiently depressed that superflow will become unstable, and the heat capacity will diverge.[2] Surprisingly, it is predicted that it will diverge far more strongly than its near-logarithmic behavior with no heat current, and at a finite value of the superfluid density.

The physics in the vicinity of this strong divergence is relatively unexplored both theoretically and experimentally. Until now, there have been no experimental studies of the heat capacity of ^{4}He under a counterflow. Although there have been a number of experiments[3,4,5,6] investigating the depressed transition temperature, there is no consensus on either the meaning of the results or the notation[7] to describe them.

2. THE TRANSITION TEMPERATURE

The depressed transition temperature, $T_c(Q)$, is expected to scale with Q as $T_{\lambda 0} - T_c(Q) \sim Q^x$, where $T_{\lambda 0} = T_\lambda(Q=0)$. Theories[8] predict that $x = 1/2\nu = 0.746$, where $\nu = 0.6705$ is the correlation length exponent[9]. Haussman and Dohm[10] (HD) have applied renormalization-group theory to the problem and obtained a quantitative prediction for the magnitude of the depression. However, in a prior conductivity experiment, Duncan, Alhers, and Steinberg[4] (DAS) observed that the onset of thermal resistance occurred at a temperature, which we will call $T_{DAS}(Q)$, below the theoretical value of $T_c(Q)$. There are two interpretations for this discrepancy.

Because the order parameter does not go to zero at $T_c(Q)$, HD[10] likened the transition to a spinodal line of a first-order phase transition. This implies that, when approaching $T_c(Q)$ from the superfluid side, fluctuations will induce the transition to occur at a lower temperature. Liu and Ahlers[5] (LA) identify this lower temperature with $T_{DAS}(Q)$. Furthermore, they report the observation of a region of small but finite resistivity that they believe lies between $T_c(Q)$ and $T_{DAS}(Q)$. Recently, Murphy and Meyer[6] confirmed the existence of this anomalous dissipative region, but called LA's placement of the region into question.

The second interpretation proposes that the difference between experiment and theory is due to the presence of a nonsuperfluid, or normal, region in the sample. As soon as an interface enters the cell, the temperature of the superfluid asymptotically approaches a unique temperature $T_\infty(Q)$ far away from the normal fluid. Originally, HD[10] hypothesized that this was the temperature that was measured by DAS. There is recent evidence to support this interpretation. Moeur, *et. al.*[11] report the observation of a self organized critical state in non-equilibrium ^{4}He. They find that for $Q > 0.5$ $\mu W/cm^2$, the temperature of this state, T_{SOC}, is in good agreement with $T_{DAS}(Q)$. A subsequent theory by

Weichman and Miller[12] proposes that T_{SOC} should occur at $T_{\infty}(Q)$. It is therefore reasonable to assume that $T_{DAS}(Q)$ and $T_{\infty}(Q)$ are identical.

The resolution of this debate is a matter of more than just theoretical interest. It also has important implications for the experimental investigation of the heat capacity under constant heat flow, as will be discussed below.

3. EXPERIMENTAL SETUP AND PROCEDURE

Measurements were taken in a cylindrical cell that consisted of two 6.985 cm diameter OFHC copper annealed endplates connected by a 0.640 mm high stainless-steel sidewall (Fig. 1). The small cell height was chosen to minimize gravitational rounding of the heat capacity, yet still be large enough to avoid finite-size effects. The cell was filled with ultra-pure ^{4}He at a temperature just below $T_{\lambda 0}$, and then sealed with a mechanical valve. Heat capacity measurements were taken at constant volume.

The temperature of the helium sample was monitored with a high resolution paramagnetic salt thermometer (HRT)[13] that provided a resolution of $5 \times 10^{-11}\, K / \sqrt{Hz}$ near $T_{\lambda 0}$. The HRT was thermally connected to the helium sample through an OFHC copper knife edged ring in pressed contact with the stainless-steel sidewall. Measuring the temperature through a sidewall probe avoids the Q-dependent Kapitza resistance[14] that would affect the temperatures of the endplates.

The constant heat current, Q, was produced by a wire heater wound around the bottom of the calorimeter (heater 1 in Fig. 1). The cell was mounted on a three-stage thermal isolation system. The third stage consisted of a radiation shield that surrounded the calorimeter. During the experiment, the temperature of the shield stage was controlled to $\pm$ 0.2 μK with another HRT.

Heat capacity measurements were taken by cooling and then heating the

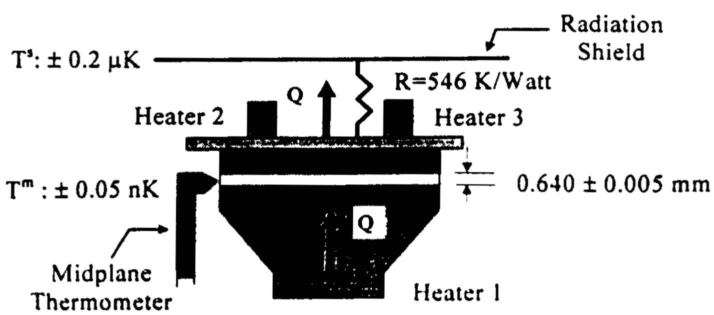

Fig. 1. Schematic diagram of the experimental cell

sample through the transition at a rate of several *nK/sec*. In order to provide a reference for our data, C_Q, we also measured the heat capacity with no heat current, $C_V(Q=0)$. The heat capacity measurements were taken under identical experimental conditions by alternately turning Q on and off for each heating-cooling sweep.

4. RESULTS

Data were taken at three different heat current values, $Q = 0.243\ \mu W/cm^2$, $1.12\ \mu W/cm^2$, and $3.58\ \mu W/cm^2$. The results are shown in Fig. 2a. The heat capacity at zero heat current was compared to established results. There were no reported data taken of C_V in the lowest reduced temperature range of our experiment. Ahlers[15] reports that at 10 μK below the transition, and at 0.05 bar, $\gamma = C_P/C_V = 1.035$. We find that at the same temperature and approximate pressure, our measurements are smaller than C_P by a factor of 1.07. We cannot currently account for this discrepancy.

Each heat capacity curve taken at non-zero heat current exhibits a sharp rise (sr) in slope at a distinct temperature, $T_{sr}(Q)$, that increases with applied Q (indicated by arrows in Fig 2a). We believe that this rise occurs when the helium at the bottom of the cell no longer has zero resistivity. We therefore associate $T_{sr}(Q)$ with $T_{DAS}(Q)$. As the temperature rises further, a portion of Q goes into creating a temperature gradient in the dissipative layer, and less heat current flows out of the top of the cell. Consequently, the magnitude of the temperature drift rate is reduced, and the heat capacity appears to be significantly enhanced. It is therefore no

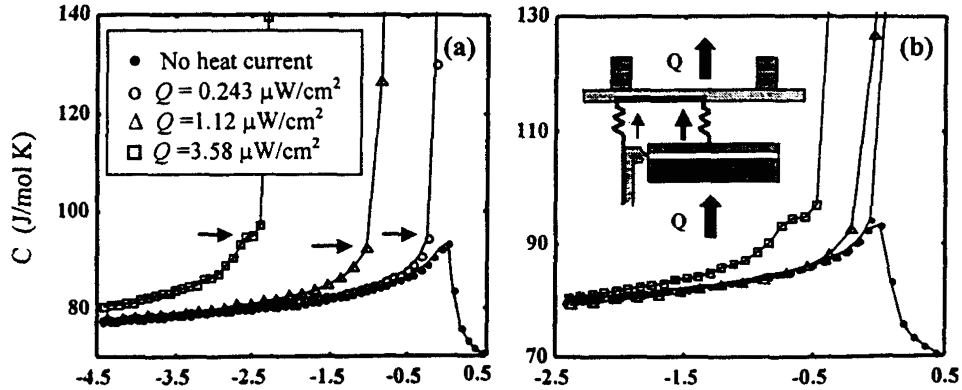

Fig 2. Heat capacity data taken at various heat currents. a) The original data. Arrows indicate where we believe dissipative fluid enters the cell. Data to the right of these points should not be considered. b) The data shifted in temperature to adjust for boundary resistance. Inset: a schematic of the thermal network responsible for the temperature offset.

longer a valid heat capacity measurement, and data at temperatures greater than this sharp increase should not be considered.

By determining the physical meaning of $T_{DAS}(Q)$, we can gauge how close our heat capacity measurements can get to the transition. In the presence of gravity, the lambda point temperature in zero heat flow varies as a function of cell height, z, due to hydrostatic pressure in the fluid. All of the transition temperatures discussed in this paper lie parallel to the line of critical points, $T^z_{\lambda 0}$. If $T^z_{DAS}(Q)$ is equivalent to $T^z_{\infty}(Q)$ (see Fig. 3a), then the cell will not contain dissipative fluid until the bottom layer of helium reaches a temperature of $T^b_c(Q)$. Up until this point, all of the helium in the cell is superfluid and uniform in temperature, and so the temperature of the midplane will also be at $T^b_c(Q)$. Afterwards, a temperature gradient interferes with the measurement. Gravity will therefore play the primary role in limiting the experiment from reaching the midplane transition temperature, $T^m_c(Q)$. If, instead, $T_{DAS}(Q)$ indicates the onset of an anomalous dissipative region below $T_c(Q)$ (see Fig. 3b), then the midplane temperature will only reach $T^b_{DAS}(Q) < T^b_c(Q)$ before a temperature gradient enters the cell. It should be noted that the former interpretation implies that an experiment with carefully designed time constants should see the temperature of the midplane thermometer drop from $T^b_c(Q)$ to $T^b_{DAS}(Q)$, before rapidly increasing as an interface moves through the cell.

Unfortunately, our present data can not resolve which interpretation of $T_{DAS}(Q)$ is correct. We found that $T_{sr}(Q)$ is considerably lower in temperature than both $T_{DAS}(Q)$ and $T_c(Q)$. We believe that this discrepancy indicates that there was spurious heat flow in our experiment.

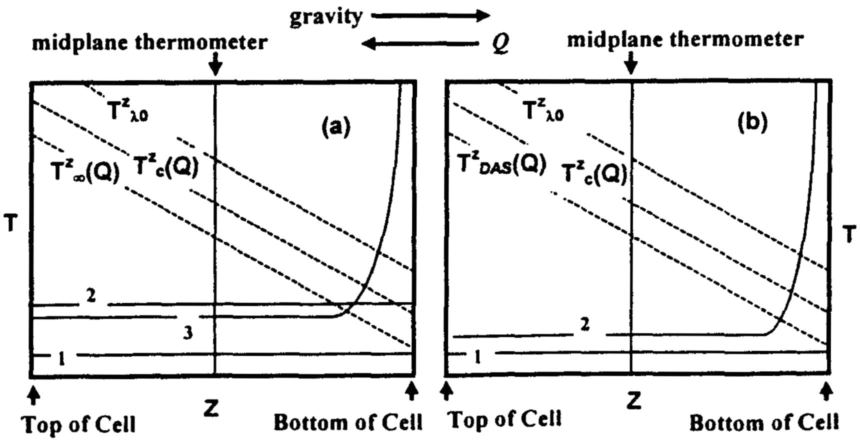

Fig. 3. Two schematic representations of thermal profiles in a cell of He^4 with a heat current under the influence of gravity. The numbers represent subsequent temperature profiles when scanning up in temperature. The thermal profiles if a) $T_{DAS}=T_{\infty}$. Line 2 is the profile at the instant the bottom of the cell reaches $T_c^b(Q)$. Line 3 is the profile afterwards. b) T_{DAS} indicates an anomalous dissipative region. At line 1, the temperature of the cell reaches $T^b_{DAS}(Q)$. Then the thermal profile indicated by line 2 develops.

Specifically, we think that a fraction of Q flowed out through the sidewall of the cell and up the niobium capillary of the HRT. This heat flow would cause a Kapitza offset in the HRT output causing the onset of dissipation to appear at a lower temperature. The difference between $T_{sr}(Q)$ and $T_{DAS}(Q)$ is a linear function of Q, lending credence to this hypothesis. The apparatus is currently being modified to eliminate this effect.

The above argument warrants a reanalysis of our data by shifting the C_Q data in temperature relative to the data taken at zero heat current. When the data are shifted so that $T_{sr}(Q)$ corresponds with $T_{DAS}(Q)$ (Fig. 2b), the four heat capacity curves overlap far away from $T_{\lambda 0}$, but become dependent on Q as the transition is approached. Although these preliminary results are promising, a quantitative analysis would be premature and further experimental data are required.

ACKNOWLEDGEMENTS

We would like to thank P. Weichman, R. Duncan, and W. Weber for helpful discussions. This research was carried out in collaboration between the Physics Dept. and the Jet Propulsion Laboratory of the California Institute of Technology under a contract with the National Aeronautics and Space Administration.

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The theory of the heat capacity of He^4 in the presence of a heat current

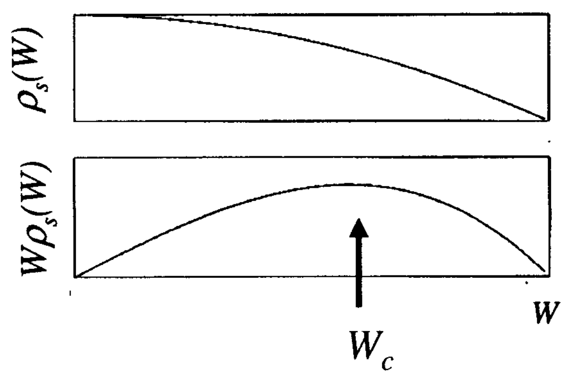
[1] A heat current induces a counterflow, W, between the superfluid and the normal fluid.

[2] This superflow depresses the superfluid density, ρ_s.

[3] If the heat current, Q, is held constant, the superfluid density will become sufficiently depressed that superflow will become unstable, and the heat capacity will diverge at a depressed transition temperature, $T_c(Q)$:

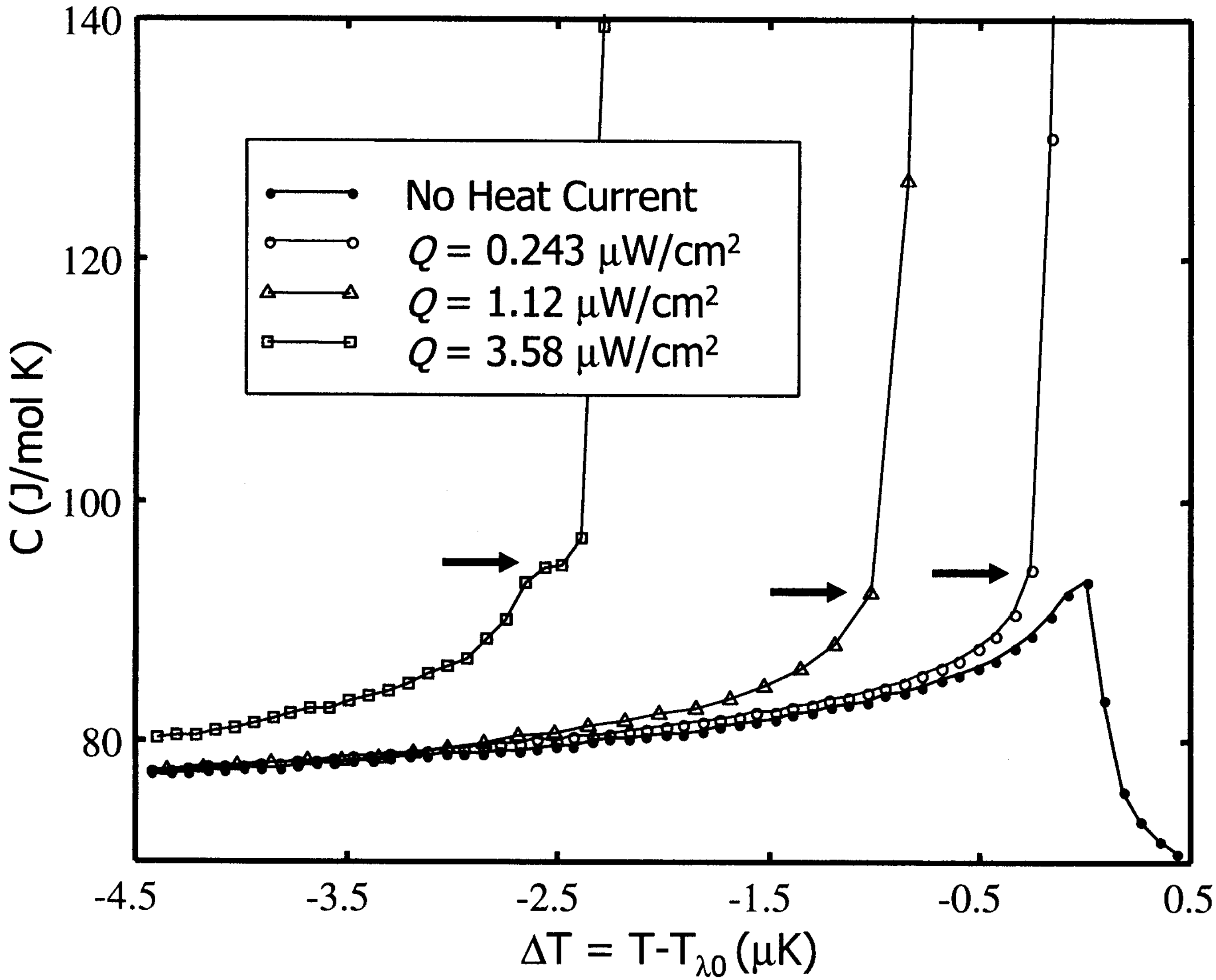
$$C_Q = C_W + TV\left(\frac{\partial P}{\partial T}\right)_W^2 \Big/ \left(\frac{\partial P}{\partial W}\right)_T$$

where $\quad P = \rho(W) \bullet W$



[4] The exponent of the divergence is 1/2: $\quad C_Q \sim \left[(T_c(Q) - T)/T_c(Q)\right]^{-\frac{1}{2}}$

Heat capacity results



Arrows indicate the temperature at which the bottom layer of helium in the cell no longer had zero resistivity.

We associate this temperature with T_{DAS}.

However, it is not equal to T_{DAS}, leading us to believe that there was a heat leak that caused a Kapitza resistance between the sidewall thermometer and the helium sample.

Data to the right of the arrows should not be considered.

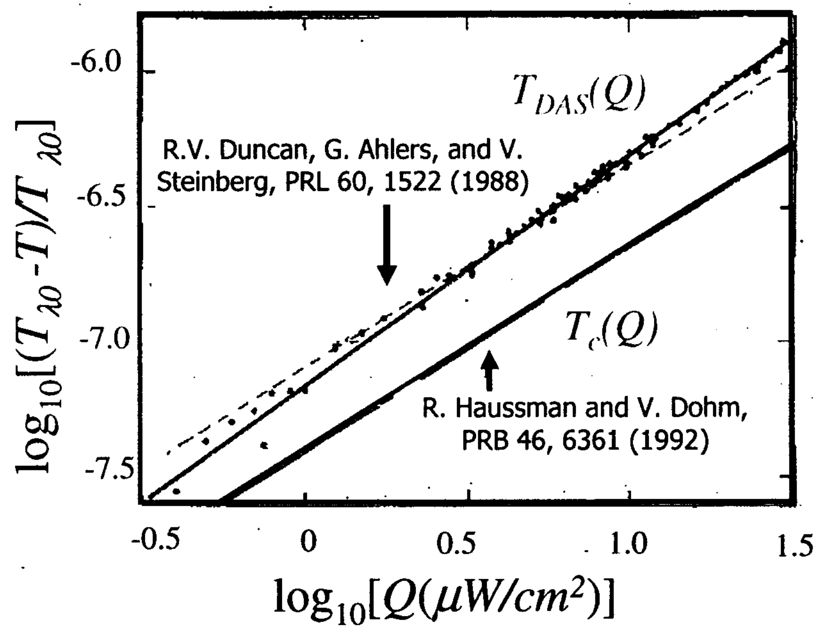
The depressed transition temperature

Theory predicts the transition temperature, T_c, scales as:

[redacted]

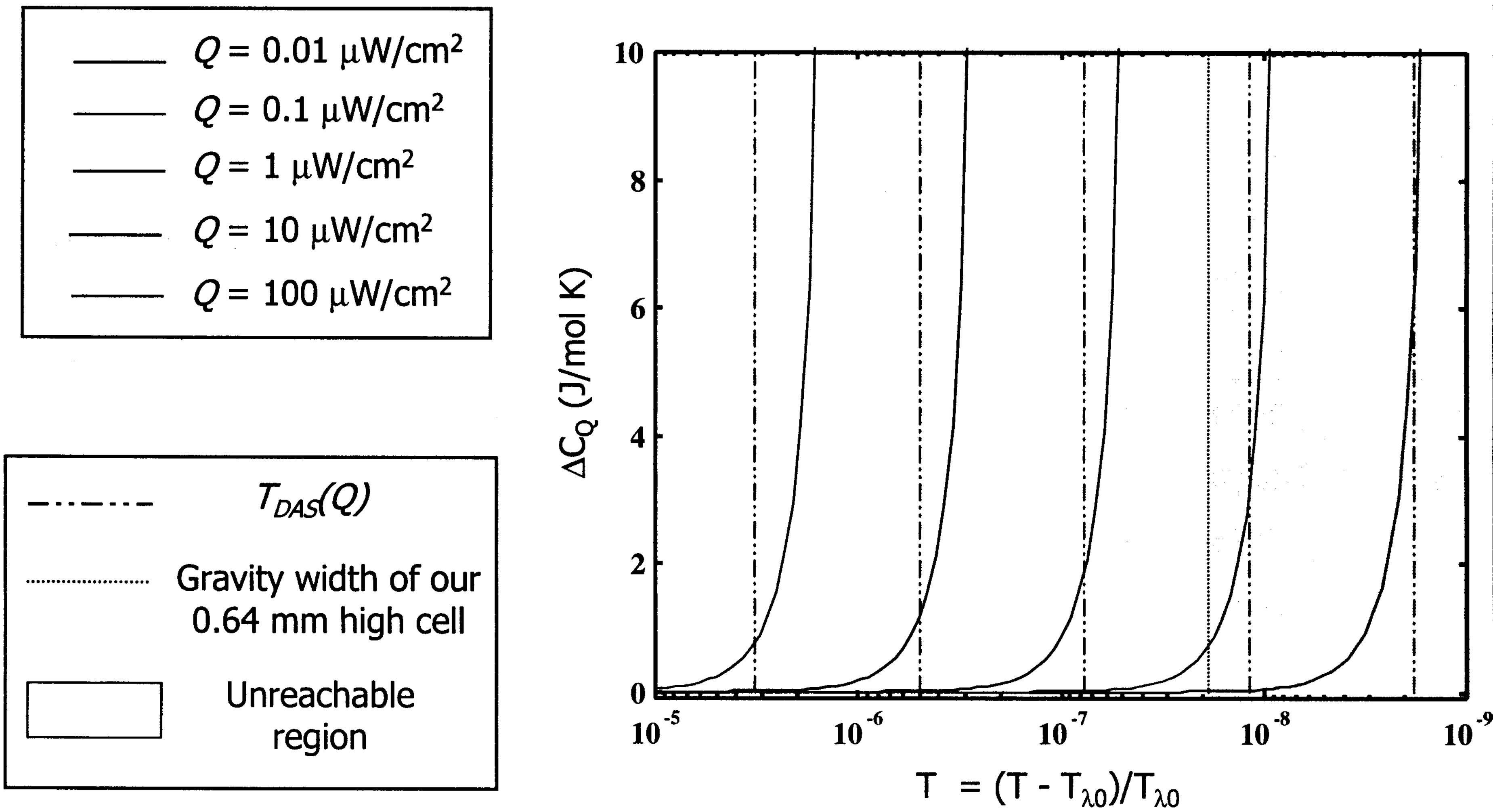
Experiment observes the onset of dissipation at T_{DAS}:

[redacted]

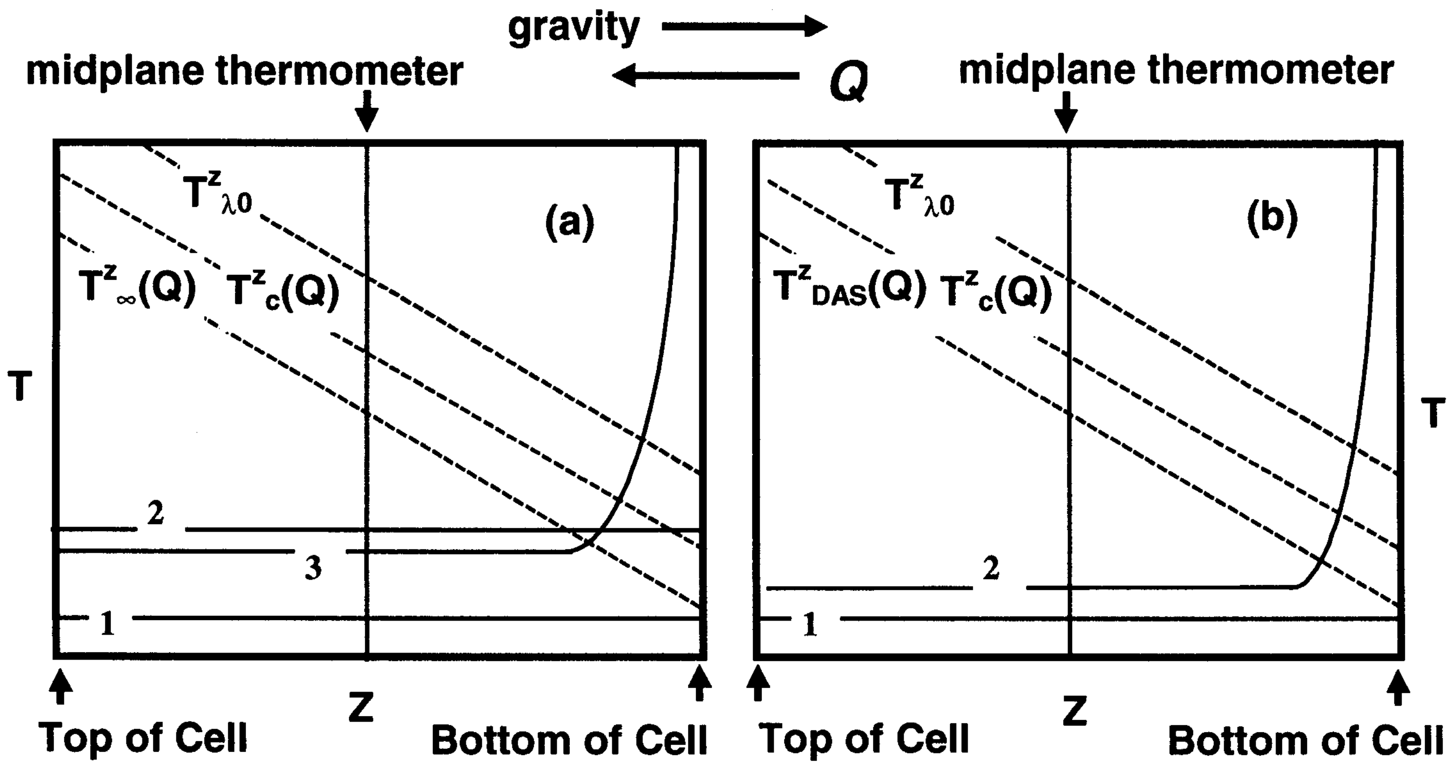


$$T_{DAS} < T_C < T_{\lambda 0}$$

Theoretical values of the change in heat capacity under a constant heat current



Two interpretations of T_{DAS}



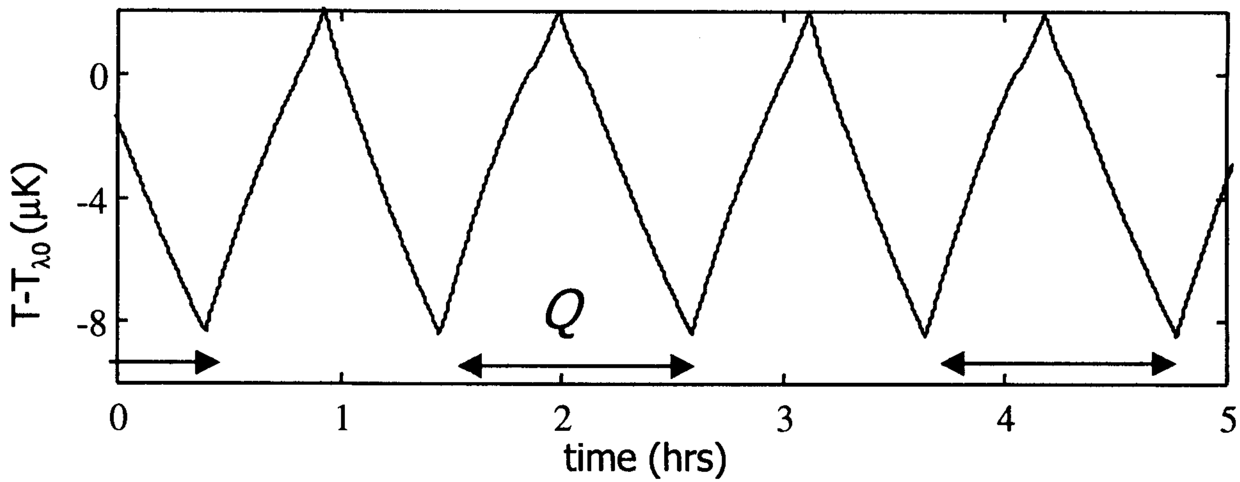
Two schematic representations of thermal profiles in a cell of He^4 with a heat current, under the influence of gravity. The numbers represent subsequent temperature profiles when scanning up in temperature. The thermal profiles if a) $T_{DAS}=T_\infty$. Line 2 is the profile at the instant the bottom of the cell reaches $T_c^b(Q)$. Line 3 is the profile afterwards.
b) T_{DAS} indicates an anomalous dissipative region. At line 1, the temperature of the cell reaches $T^b_{DAS}(Q)$. Then the thermal profile indicated by line 2 develops.

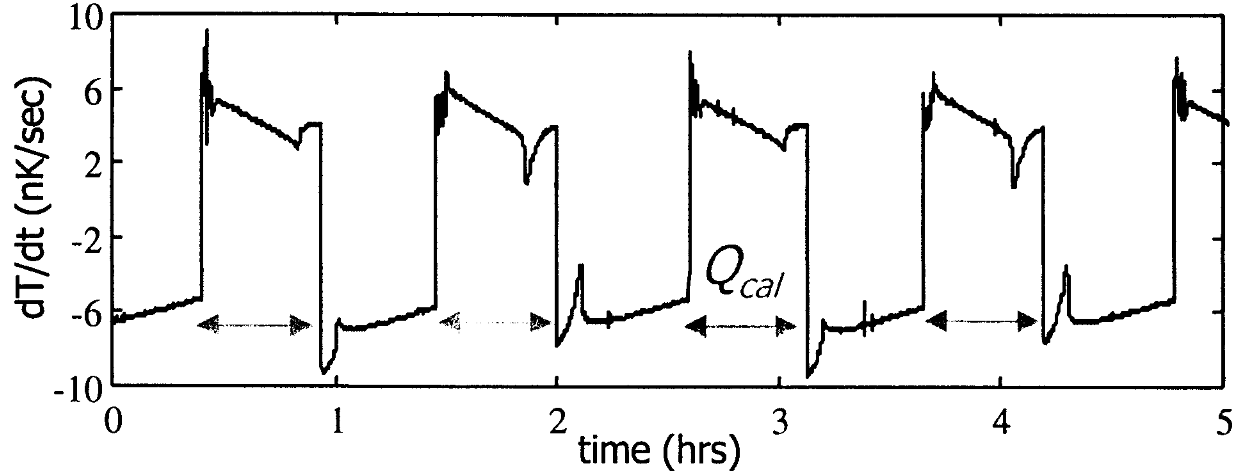
Experimental procedure

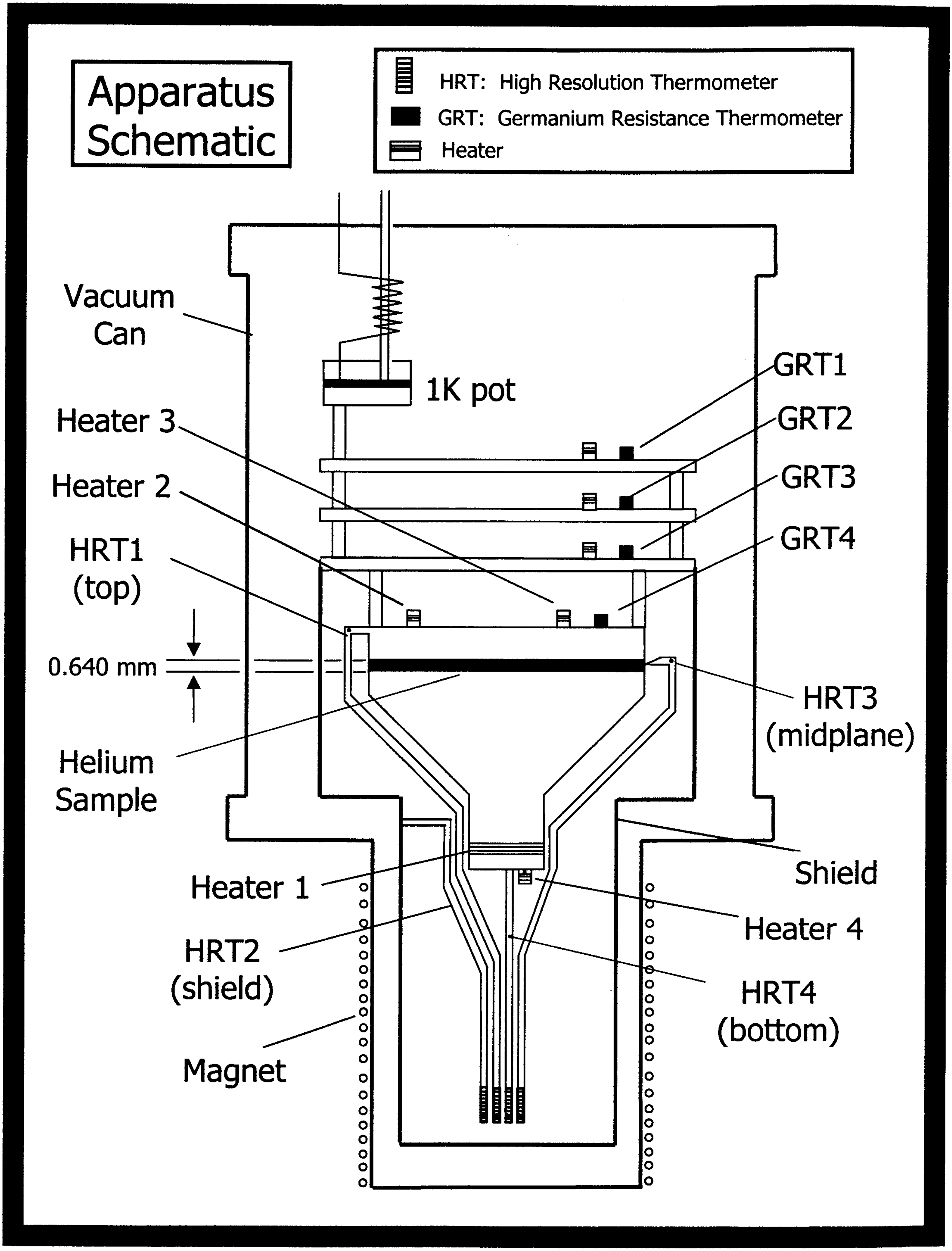
- Set the shield temperature such that, with Q applied, the temperature of the helium sample drifts down at several *nK/sec.*
- Allow the temperature to drift down through the transition to a minimum temperature located 5-10 μK below $T_c(Q)$.
- Turn on a calorimetry heat current, Q_{cal}, such that the temperature drifts upward at several *nK/sec.*
- Allow the temperature to drift up through the transition to a maximum temperature located several μK above $T_c(Q)$.
- Turn off Q_{cal}.
- Repeat. At the bottom of every upward drift, switch Q on or off so that C_Q and $C_V(Q=0)$ are taken on alternating sweeps.

$$C(T)=Q_{cal}\Big/\left[\left(\frac{dT}{dt}\right)_{up}-\left(\frac{dT}{dt}\right)_{down}\right]$$

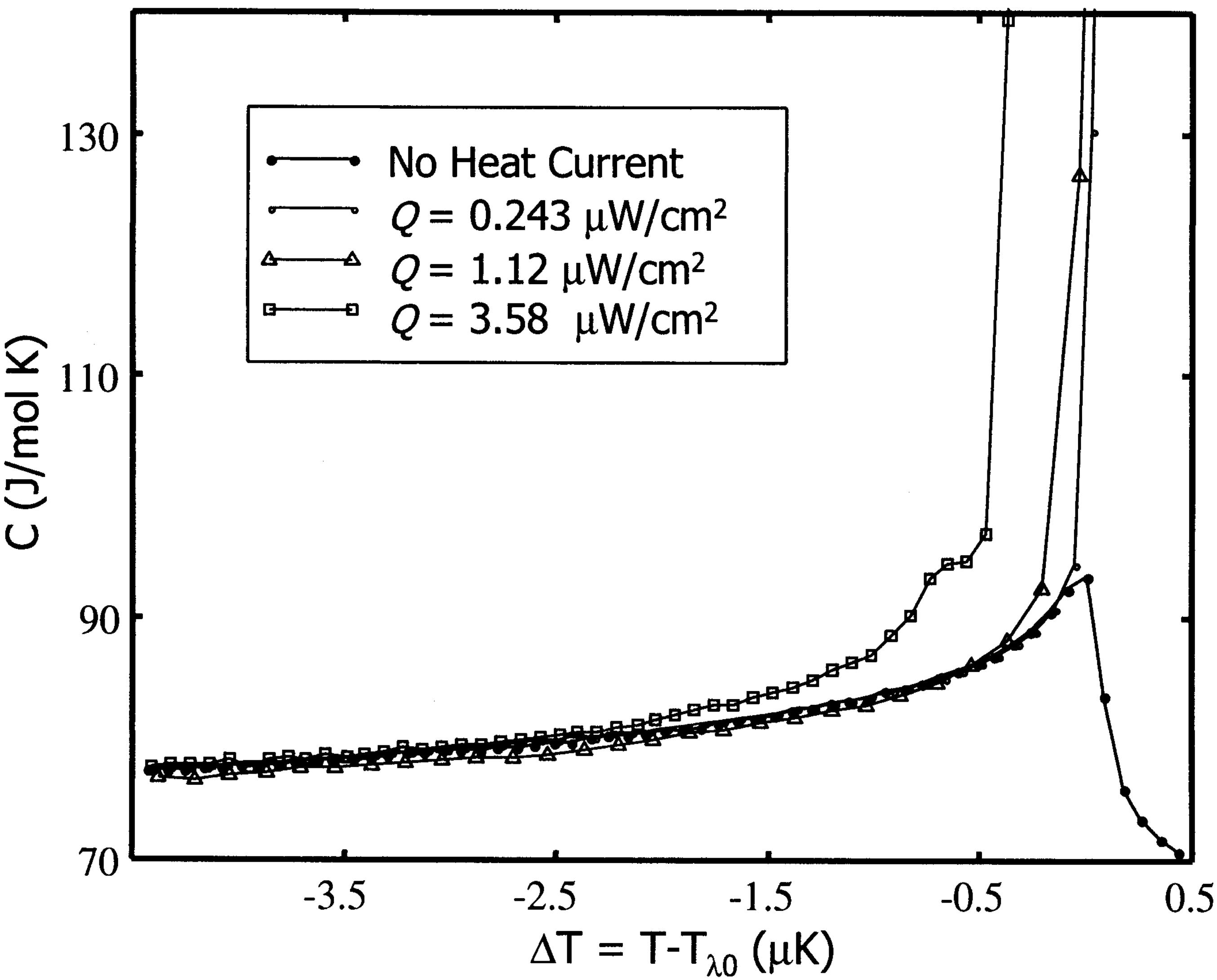
$Q = 0.243\ \mu W/cm^2$






Apparatus Schematic
HRT: High Resolution Thermometer
GRT: Germanium Resistance Thermometer
Heater
Vacuum Can
1K pot
GRT1
GRT2
GRT3
GRT4
Heater 3
Heater 2
HRT1 (top)
0.640 mm
HRT3 (midplane)
Helium Sample
Shield
Heater 1
Heater 4
HRT2 (shield)
HRT4 (bottom)
Magnet

Shifted heat capacity results



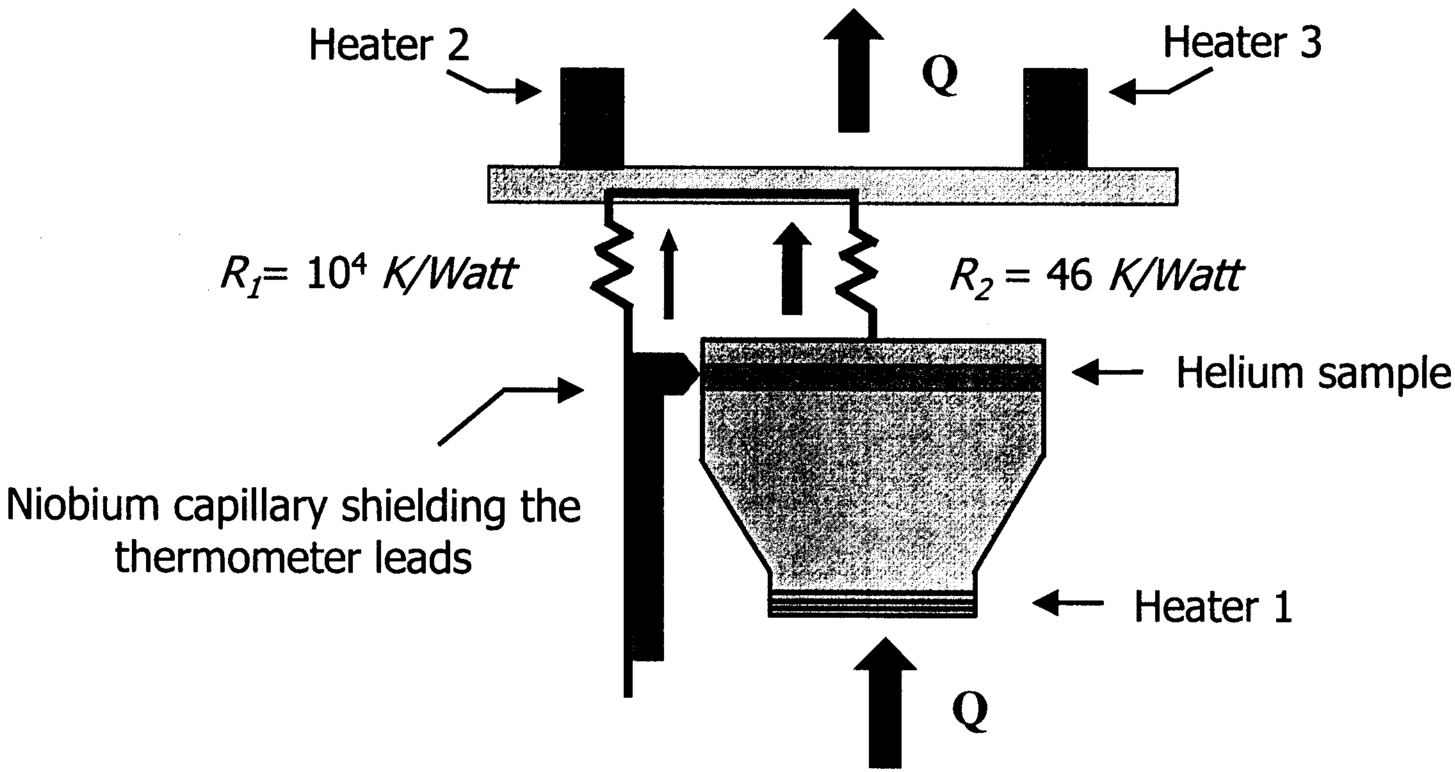
The C_Q data has been shifted in temperature so that $T_{DAS}(Q)$ is aligned with the temperature at which dissipation enters the bottom layer of the cell.

The magnitude of the temperature shift is a linear function of Q.

In agreement with theoretical predictions, the data overlap at temperatures far below $T_{\lambda 0}$, but become dependant on Q as the transition is approached.

The cause of the temperature shift in our data

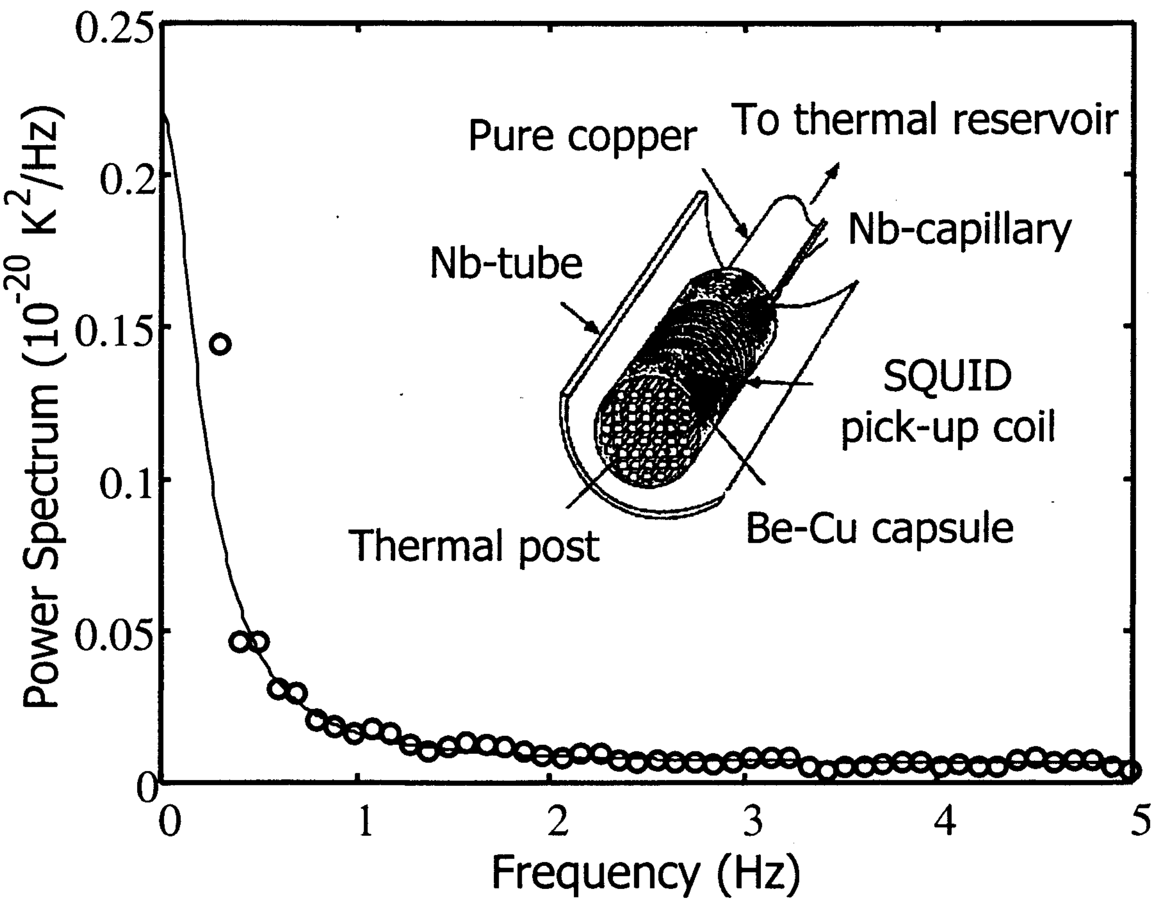
The niobium capillary shielding the leads of our midplane thermometer caused a heat leak through the sidewall of the helium cell. This heat flow led to a Kapitza resistance between the helium and the thermometer that resulted in a temperature shift of our data as a function of heat current.



High Resolution Thermometers

In its basic form, an HRT consists of a magnetic salt tightly coupled to a superconducting pick-up coil. The pick-up coil is connected to a SQUID that measures the changes in magnetization of the salt as a function of temperature. The chosen salt has a Curie temperature close to the temperature range to be measured, so that its magnetization is highly temperature dependent. The HRTs used in this experiment were constructed with $GdCl_3$ doped with Lanthanum and have a Curie Temperature of 2.185 K.

Resolution : $5\times10^{-11}\ K/\sqrt{Hz}$



Conclusion

When the data are adjusted so that $T_{DAS}(Q)$ lines up with the temperature at which a dissipative layer enters the cell,

[1] All of the heat capacity curves overlap at temperatures far below $T_{\lambda 0}$

[2] As $T_{\lambda 0}$ is approached, the heat capacity becomes dependant on Q

[3] Although these preliminary results agree qualitatively with theoretical predictions, more experimental data are required.